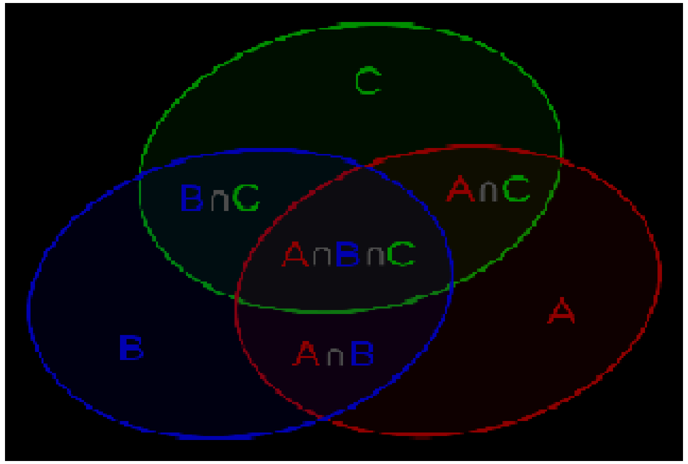
The Inclusion and Exclusion Principle

- (i) $n(A\cup B) = n(A)+n(B)-n(A\cap B)$

- (ii) $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$



Q.In a Class of 75students the number of students who passed in mathematics is equal to the number of students who passed in electronics . The total number of students who passed exactly in one subject is 60. it was found that 5 students did not pass in either subject. Find the the number of students who passed in mathematics only, who passed in electronics only and who passed in both subjects.

Let M and E denote the set of students who passed in Mathematics and electronics respectively

Further , n(M)=n(E)=x(say)and also $n(M\cap E) = y$

$n(M\cup E) = n(S) - n(\overline{M \cup E})$ $= n(S) - n(\bar{M} \cap \bar{E})$

So $n(M\cup E)=75-5=70$

Further, $n(M\cup E)=n(M)+n(E)-n(M\cap E)$

70=x+x-y OR 70=2x-y …………..1

The total number of students who passed exactly in one subject is 60

Only Pass in Mathematics=n(M)-n($M\cap E$)

Only Pass in Electronics =n(E)-n($M\cap E$)

Given Only Pass in Mathematics+Only Pass in electronics=60

So n(M)-n($M\cap E$)+n(M)-n($M\cap E$)=60

x-y+x-y=60 or 2x-2y=60 or x-y-30 2

on solving1 and 2 , x=40, y=10

Only Pass in Mathematics=n(M)-n($M\cap E$)=40-10=30

Only Pass in Electronics =n(E)-n($M\cap E$)=40-10=30

Passed in both subjects=10

Q. In the class, 42%students passed in mathematics ,45%passed in physics,41%passed in chemistry, 16%passed in mathematics and physics ,19%passed in physics and chemistry,18% passed in chemistry and mathematics, find then number of students who passed in alll the three subjects if there were 260 students in the class and 15%students failed in all subjects .

Solution: We are given the following %i.e.the number of students passing out of 100.

$n(M)=42$, $N(P)=45$, $N(C)=41$, $n(M\cap P)=16$, $n(P\cap C)=19$, $n(C\cap M)=18$

And we want $n(M\cap P\cap C)$, we have

$$n(M\cup P\cup C)= n(M)+N(P)+N(C)-n(M\cap P)- n(P\cap C)- n(C\cap M)+n(M\cap P\cap C)$$

$$=42+45+41-(16+19+18)+n(M\cap P\cap C) \qquad \ldots\ldots..1$$

Since 15% students failed in all the three subjects so

n(M∪P∪C)(pass in three subjects)=n(S)-($\bar{M} \cap \overline{P \cap} \bar{C}$) or
n(M∪P∪C)=100-15=85

So from 1

85=128-53+n(M∪P∪C)

n(M∪P∪C)=10%

So number of students who passed in all three subjects

=260× $\frac{10}{100}$=26

Q. A survey of 500 television watchers produce the following information:285 watch footwall ,195watch hockey,115 watch basketball,45watch footwall and basketwall,70watch footwall and hockey ,50 watch hockey and basket ball and 50donot watch any of the three kinds of game.

(a)How many people in the survey watch all three kinds of games

.

(b) How many people watch exactly one of the sports.

Ans ; Given that n(S)=500, n(F)=285,n(H)=195,n(B)=115

n(F∩B)=45, n(F∩H)=70, n(B∩H)=50 , $n(\bar{F} \cup \bar{B} \cup \bar{H})$ =50,

Number of students who all three games

$n(F\cup B \cup H)=n(S)- n(\bar{F} \cup \bar{B} \cup \bar{H}) =500-50=450$

Using addition rule

$n(F \cup B \cup H) = n(F)+n(B)+n(H)-n(F \cap B)-n(B \cap H)- n(F \cap H)+n(F \cap B \cap H)$

$450=285+195+115-70-50-45+ n(F \cap B \cap H)$

So $n(F \cap B \cap H) =20$

The number of people who watch all three kinds of games is 20

Let F_1 denote the set of people who watch only football, B_1 denote the set of people who watch only basketball, H_1 denote the set of people who watch only Hockey.

The Number of people who watch only football

$n(F_1)=n(F)- n(F \cap B)- n(F \cap H)+ n(F \cap B \cap H)$

$=285-70-45+20=190$

The Number of people who watch only Hockey

$n(H_1)=n(H)-n(B\cap H)-n(F\cap H)+n(F\cap B\cap H)$

$=195-70-50+20=95$

The Number of people who watch only Basketball

$n(B_1)=n(B)-n(F\cap B)-n(B\cap H)+n(F\cap B\cap H)$

$=115-45-50+20=40$

Number of people who watch exactly one of the sports is

$= n(F_1)+n(H_1)+n(B_1)$

$=190+95+40$

$=325$

Q Find the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2,3, or5

Ans : let the sets A divisible by by 2, B sets divisible by 3, C set divisible by 5.Then

n(A)=50 (1 to 100 half even[divisible by 2] or half odd) similarly

n(B)=33(divisible by 3) , n(C)=20(divisible by 5) ,

n(A∩B)=16 (divisible by 6),

n(A∩C)=10(divisible by 10),

n(B∩C)=6 (divisible by 15),

n(A∩ B ∩C)=3 (divisible by 30)

Number of integers divisible by 2 or 3 or 5 is

$N(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 50 + 33 + 20 - 16 - 10 - 6 + 3 = 74$

Number of integers not divisible by 2 or 3 or 5 is

$= n(S) - n(A \cup B \cup C)$

$= 100 - 74$

$= 26$

Ordered Set: A set of elements such that each element is assigned a position is called an ordered set.An ordered set with n elements is called an n-tupple
example :the number ‘3456’ is an ordered set of digit 3,4,5,6
The word ‘product’ is an order set of the letters p,r,o,d,u,c,t

The Pigeonhole principle:

Theorem 1: If n pigeons are assigned to m pigeonholes, then at least one pigeonhole contains two or more pigeons(m<n).

Ans: if we assign 1 pigeon to 1 pigeon hole and substiquainly . After m pigeon holes n-m are left for pigeon hols so we put second pigeon to a pigeon hole.

In other word “ if n pigeons holes are occupied by n+1 or more pigeons, then at least one pigeon is occupied by more then one pigeons.

Example: in a class of 101 students there must be at least two students scoring same marks in a paper having maximum marks 100.

Ans:The number of possible scores a student can be assigned is a from 0 to 100 hence total of 101 score assigned as 0,1,2,3……….100 to 101th student Now when 102 th students is given a score that will we surely repeated(0-100). Hence at least102 students should we there.

Q. In a department of 13 teachers, two of teachers were born in the same month.

Ans. the teacher(pigeons) 1- 12 were born jan to dec in12 month respectively. Last 13th teacher must born are also jan to dec months so at least to teacher were born in the same month.

Theorem 2:

If a pigeons are assigned to m pigeonhole, then one of the pigeonhole contain at least{$[\frac{n-1}{m}]$+1}pigeons, where $[\frac{n-1}{m}]$ denotes the largest integer less then and equal to the rational number.

Example: show that if 30 dictionaries in a library contain a total of 61327 pages then one of the dictionaries must have at least 2045 apges.

Ans. Pigeon are 61327 and pigeonhole are 30 then

By $\{[\frac{n-1}{m}]+1\}$ then $\{[\frac{61327-1}{30}]+1\}$ =2045pages

Q. Show that 20 persons are selected for presenting a cultural programme , then one may select a subset of 3 so that all 3 would be able to present their programme on the same day of the week.

Ans. Assign each person to the day of the week on which he would present his programme . The 20 person (pigeons) in the way are being assigned to 7 pigeonhole (days of week). Hence by pigeonhole's principle at least

$\{[\frac{n-1}{m}]+1\}= [\frac{20-1}{7}]+1=3$ nof the persons must present their programmes on the same day of the week.